TOWERS PERRIN

Developing a Hedging Program

Focused on Dynamic Hedging for Annuities

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Presentation outline

- Hedging Overview
- Dynamic Hedging Process
- Common Forms of Hedging Analyses

Hedging Overview

- Presentation focus
- Hedge objectives
- Hedge methodologies
- Hedging strategy details

Presentation focus

- The largest recent hedging focuses have been on annuities
  - Variable annuity riders
  - Equity indexed annuities
  - Fixed annuity guarantees
- All products with market risk can be actively hedged
- Dynamic hedging is key, as many companies feel in house dynamic hedging programs will result in the largest cost savings

Hedging objectives

- Minimize volatility of results
  - Stability or stop loss
  - Multiple income reporting bases
    - — GAAP valuation may not include fair value accounting
- Decrease point in time balance sheet items
  - Reserves and capital
- Stabilize embedded economic value
- Create an optimal risk versus reward tradeoff

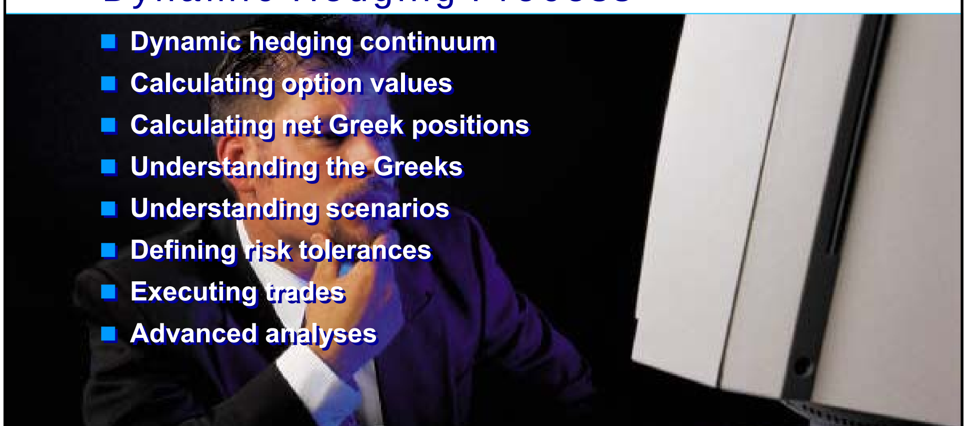
Hedging methodologies

- Reinsurance
  - Partial with caps and floors
  - Total reinsurance
- Static hedging
  - Simple exchange traded options
  - Customized structured products
- Dynamic hedging
  - Monitor market exposures
  - Calculate the greeks and execute trades
- Product design
  - Integrating hedging strategy and product design process
  - Minimizing risk in products

What is a hedging strategy?

- Defines what will be hedged and with what instruments
- Set maximum positions and a clear process for redefining maximum positions and strategies
- Defines what is not hedged and what risk remains
- Assigns clear responsibilities with appropriate controls and monitoring systems
- Define metrics, criteria, and frequency for measuring hedge effectiveness
- All routinely reviewed by supervisory management and internal or third-party auditors

Dynamic Hedging Process



- **Dynamic hedging continuum**
- **Calculating option values**
- **Calculating net Greek positions**
- **Understanding the Greeks**
- **Understanding scenarios**
- **Defining risk tolerances**
- **Executing trades**
- **Advanced analyses**

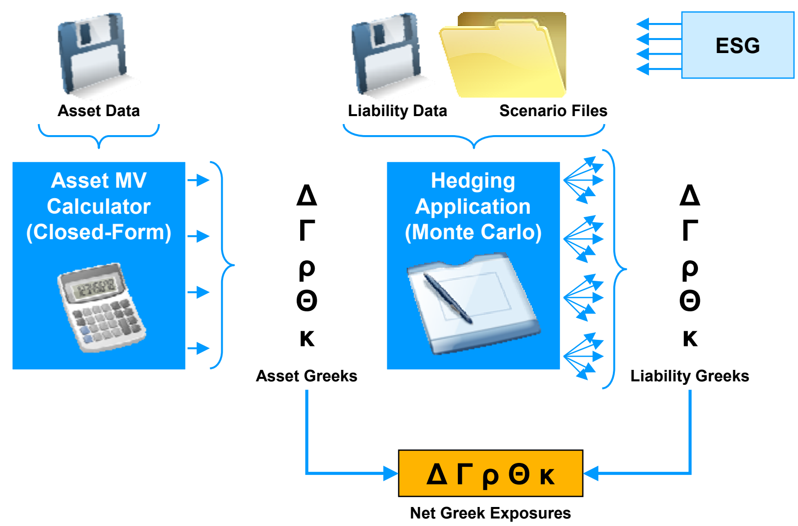
Dynamic hedging continuum

- No hedging
- Periodic monitoring
- Real time monitoring
- Creating a hedging strategy
- Mock testing
- Single-Greek hedging
- Attribution analysis and refining hedging strategy
- Multi-Greek and multi-product hedging

- Operational requirements
  - Information technology
  - Knowledgeable staff

Hedge program functional schematic

General process flow: The Greeks



Calculating option values

- Liability option values are generated via Monte Carlo simulation
  - Average of discounted projected cash flows under many different risk-neutral scenarios
- Asset option values often have closed-form solutions
- Option values are required under a variety of economic shocks
  - Efficiency is required due to the number of runs needed
  - Shocks X Scenarios X Model Points
- Option values are expressed as OV(x) in this document
  - OV indicates a baseline option value (unshocked)
  - OV(+1) indicates an option value with a shock up one unit
  - OV(-1) indicates an option value with a shock down one unit
  - Shocks can be asset prices, interest rates, or volatility rates

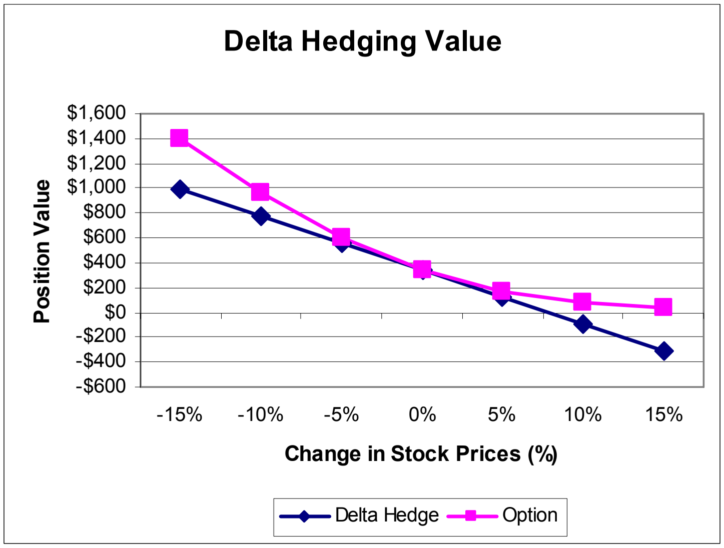
Understanding the Greeks

- Delta
- Gamma
- Rho
- Vega
- Theta
- Rho Convexity

Delta

- Measures the rate of change of an option price with respect to the price of the underlying asset ($\Delta = \partial V / \partial S$)
- Delta is calculated as follows (using asset price shocks):
  - Dollar Delta = OV(+1) – OV(-1)
- Delta will measure the expected change in value of an instrument for a small change in price. Delta will less accurately measure the expected change in value for larger changes in price
- Delta is often actively hedged for VAs, typically using equity futures
  - Highly liquid, deep market keeps transaction costs low
- Calculated for equity funds, not bond funds

Delta (cont'd)



Gamma

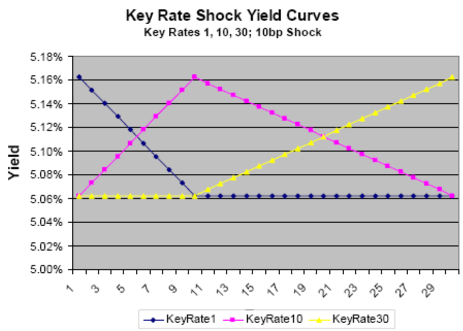
- Measures the rate of change of an option's delta with respect to the price of the underlying asset (Δ = ∂2V / ∂S2)
- Gamma is calculated as follows (using asset price shocks):
  - Dollar Gamma = OV(+1) – 2 * OV + OV(-1)
- A high Gamma suggests:
  - A high level of variation in Delta, which may then require more frequent trading to maintain low net exposure to Delta
  - Hedge ineffectiveness due to tracking error is more likely to be large with large market movements
- Gamma is less often actively hedged
  - Expensive to hedge; typically hedged with equity options
  - Trades may be made periodically to shore up Gamma exposure

Rho

- Measures the rate of change of an option price with respect to interest rates ($\Delta = \partial V / \partial r$)
- Rho is calculated as follows (using interest rate shocks):
  - Dollar Rho = OV(+1) – OV(-1)
- Rho will measure the expected change in value of an instrument for a small change in interest rates. Rho will less accurately measure the expected change in value for larger changes in price
- Rho is sometimes actively hedged for VAs, typically using swaps
  - More significant for longer exposures (ex GMWB-for-life)
- Calculated several different ways (see next slide)

Key rate Rhos

- Yield curve shocked multiple times with shock graded such that combination of all incremental key rate shocks represents a parallel shock
- Produces multiple price sensitivities, leading to the ability to hedge using multiple instruments



Vega

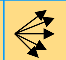
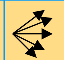
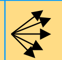
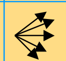
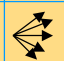
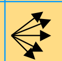
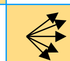
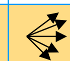
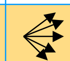
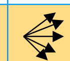
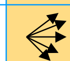
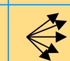
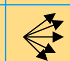
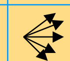
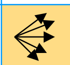
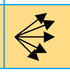
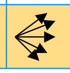
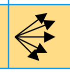
- Measures the rate of change of an option price with respect to volatility ($\Delta = \partial V / \partial \sigma$)
- Vega is calculated as follows (using volatility rate shocks)
- Dollar Vega = OV(+1) – OV(-1)
- Vega will measure the expected change in value of an instrument for a small change in volatility. Vega will less accurately measure the expected change in value for larger changes in price
- Vega is not often actively hedged
  - Cannot hedge Vega with futures; must use options (expensive) or volatility derivatives
  - Trades may be made periodically to shore up Vega exposure
- Vega exposure sometimes bucketed like Rho

Other Greeks

- Theta
  - Measures the rate of change of an option's value with respect to time ($\Delta = \partial V / \partial t$)
  - This is not traditionally hedged
- Rho Convexity
  - Measures the rate of change of an option's rho with respect to the price of the underlying interest rate ($\Delta = \partial 2V / \partial r2$)
  - Often monitored but not actively hedge
  - A large rho convexity can predict larger ineffectiveness with big interest rate moves

Traditional approach to calculating Greeks

ILLUSTRATION

Greek	Parallel	Equity / Bond				Key Rate			
		Fund 1	Fund 2	...	Fund N	Rate 1	Rate 2	...	Rate N
Delta									
Gamma									
Rho									
Rho Convexity									
Vega									


Represents appropriate number of shock processes required to calculate the Greek in question.

Greek calculation requires scenarios

- Calculation of the Greeks requires a risk-neutral valuation of assets and liabilities
- Liabilities should be valued in same fashion actively-traded financial instruments are valued
- A calibrated, risk-neutral scenario file is needed for the baseline projection and some of the extra shocks
- Poorly calibrated risk-neutral scenario sets unable to effectively reproduce market prices will result in:
  - Misestimation of the Greeks
  - Execution of inappropriate trades
  - Greater hedge ineffectiveness / slippage

Scenario requirements

- Creation of a quality risk-neutral scenario set requires:
  - Access to market data
    - Swaption, equity volatilities
    - Interest rates
    - Correlations
  - Careful choice of market surrogates
    - Can I get good market data on the surrogates I chose?
  - Enough time to support hedging processing

Prospect of calculating the Greeks so massive sometimes RN scenario generation effort and requirements are underestimated

ESG scenario generation process flow

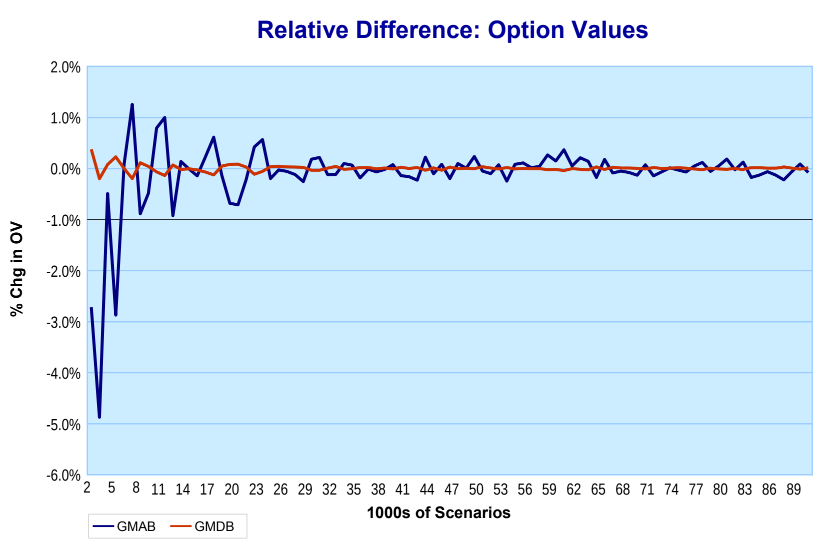
Calibration		RN Generation
Objective: Parameterize the RN generator such that market prices can be replicated (or closely replicated) under a variety of market circumstances	→	**Objective:** Produce a set of RN scenarios that will reproduce market prices using the calibration parameters supplied in the calibration stage
Inputs: Current market data, including yields, swaption volatilities, equity volatilities, correlation assumptions	→	**Inputs:** Calibration parameters

Ensuring quality of RN scenario set

Several questions should be asked after creation of a risk-neutral scenario set to be used for hedging purposes:

- Is the set in fact risk-neutral?
- Will the set reproduce market prices?
  - Use the scenario set to reproduce market prices entered into calibration process
- Will the scenario set produce stable option values?
  - This is another way of asking have I generated a sufficient number of scenarios to produce stable option values?
  - Convergence testing often employed

Convergence testing example



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Finishing steps

- Defining risk tolerances
  - Maximum net Greek tolerances

Greek Limits	Delta	Gamma	Vega	Rho			Ccy
Total	22	0.028	0.6	1.5			USD MI
Vega Bucket Limits	0 – 6m	6m – 1y	1y – 2y	2y – 3y	3y+	Total	Ccy
Total	0.13	0.35	0.4	0.3	0.5	0.6	USD MI
Rho Bucket Limits	0 – 6m	6m – 1y	1y – 2y	2y – 3y	3y+	Total	Ccy
Total	0.4	0.4	0.55	0.75	1.4	1.5	USD MI

- Use defined hedging instruments to keep all Greeks within risk tolerances
- Monitor and revise hedging as necessary

Common Forms of Hedging Analyses

- **Greek quantification**
- **Projection capabilities**
- **Attribution analysis**

Common forms of hedging analyses

Greek quantification

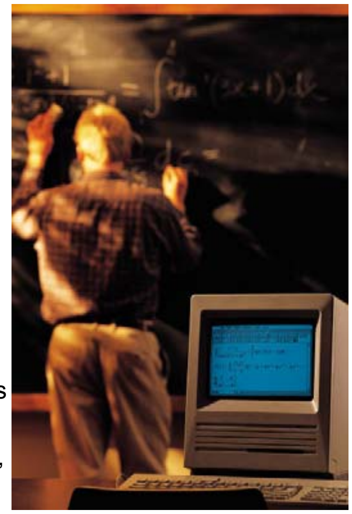
- Calculate exposure at a point-in-time
- Helps administer operational side of hedging program

Projection capabilities

- Ability to perform 'what-if' analysis on a block of business using various hedging strategies / techniques
- Helps assess capital requirements, earnings volatility, hedge effectiveness

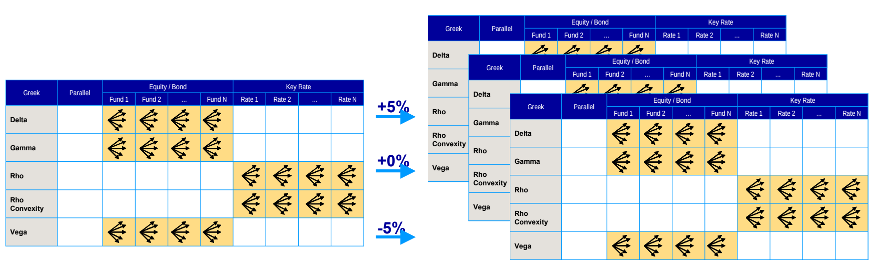
Attribution analysis

- Attributes P&L gain/loss to various factors
- Helps management understand why hedging strategy not completely effective, what drivers are



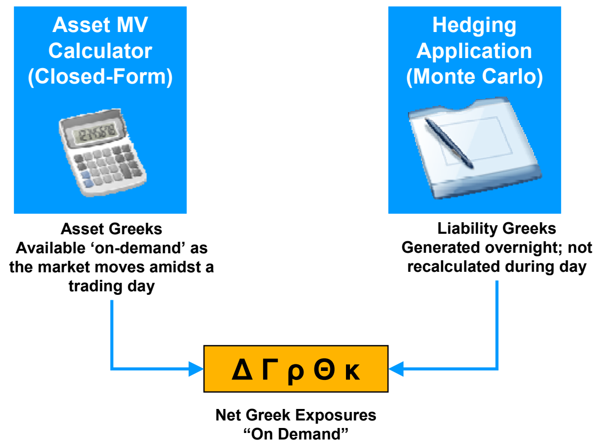
Greek quantification

- Greek grid was presented as two-dimensional with a locus at current market prices
- To maximize effectiveness throughout a trading day, some companies will add another dimension for market locus
- Will make company more responsive to dramatic changes



Greek quantification (cont'd)

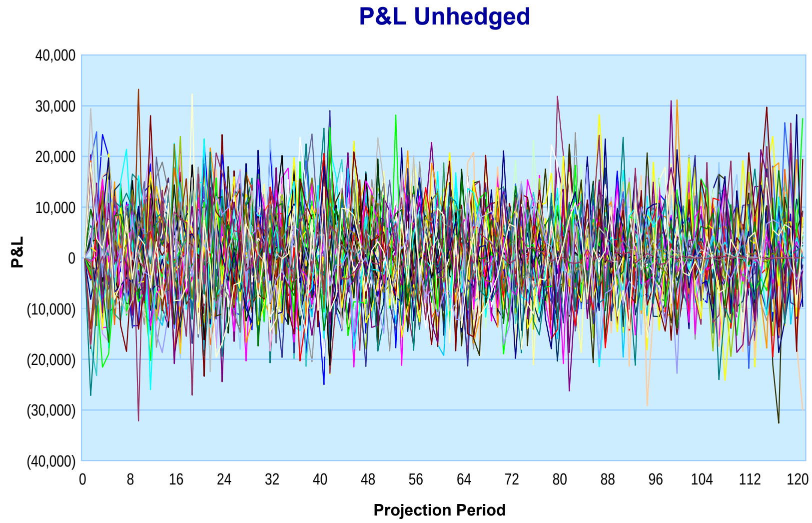
- Now company has a wider range of liability Greeks to draw from amidst trading day



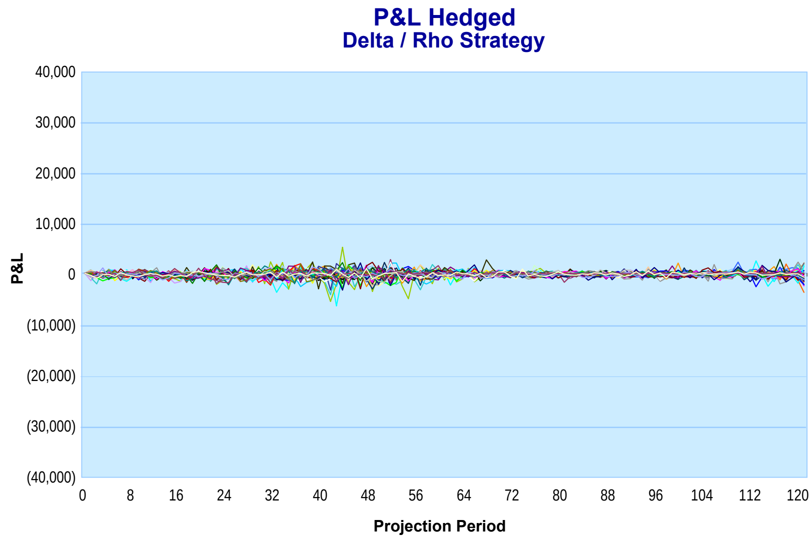
Projection capabilities

- Financial models service a wide range of actuarial functions
- Treatment of hedging is often omitted or greatly approximated in these models
- Explicit modeling of dynamic hedging strategies provides great insight into the following for various hedging strategies:
  - Earnings volatility
  - Capital requirements
- Can help management understand why hedging program has not completely eliminated all risk
- Can test the impact of new hedging strategies

Projection capabilities: Earnings volatility analysis



Projection capabilities: Earnings volatility analysis (cont'd)



Attribution analysis

- Attribution analysis attempts to attribute the hedge ineffectiveness observed between two points in time to various factors
- Economic gains/losses attributed to:
  - Market movements (delta, vega, rho, gamma, basis)
  - Liability experience (lapses, mortality, withdrawals, transfers)
  - Elapsed time
- General process:
  - Calculate expected economic gain/loss
  - Incrementally add actual experience from various sources; requires option revaluation for each contributing factor
  - Compare end result to actual economic gain/loss
  - May have residual amount