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Assessing volatility and credibility of experience – a comparison of approaches

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Agenda

Volatility

- Its definition
- Its importance
- How to measure it

Credibility

- Its definition
- Its importance
- Its relationship to volatility
- How to calculate it and apply it

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Perspective

- Examples here revolve around mortality experience
- Could be translated and applied to other types of experience that are subject to volatility and/or credibility
  - Medical
  - Disability
  - Critical illness
  - Property/casualty
  - Slot machines

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ˌvä-lə-ˈti-lə-tē\ *noun*

- Definitions of volatility, according to www.merriam-webster.com:

1. Ability to readily vaporize at a relatively low temperature
2. Having the power to fly
3. The tendency to erupt into violence
4. Inability to hold the attention fixed because of an inherent lightness or fickleness of disposition
5. Subjectivity to rapid or unexpected change

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Blame it all on volatility

- Shareholder confidence
- Stock price
- Your bonus
- Assumption setting difficulty
- Workday disruptions
- Your boss's confidence
- Your job security
- The weather

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Common measures of volatility

- Risk management
  - "200-year event" = 99.5th percentile
  - "20-year event" = 95th percentile
- Relative standard deviation: RSD = StDev / Mean
- Percentiles of aggregate claims distribution
- Confidence intervals

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What's the bottom line?

- Results by number of claims
  - Easier to calculate volatility
  - Not always the answer to the question being asked
- Results by amount of claims
  - More difficult to estimate volatility but not impossible
  - Here's your bottom line
  - No wait, this is

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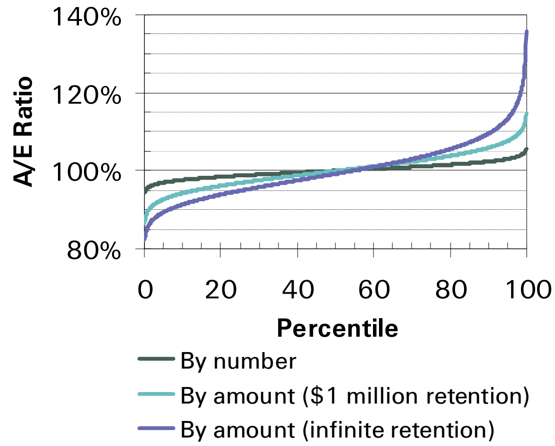
Aggregate claims distribution

- Graph of the percentiles of an aggregate claims distribution:



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Aggregate claims distribution: Two approaches

Monte Carlo simulation

The Panjer method

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Aggregate claims distribution: Two approaches

Monte Carlo simulation

The Panjer method

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About Monte Carlo simulation

- Intuitive approach
  - Formulas are fairly simple
  - Easy to draw parallels to related processes like flipping coins or rolling dice
  - Interpretation of results is straightforward
- Can use exact amounts (“bucketing” not necessary)
- Computationally intense
  - 10,000 Monte Carlo trials on a block of 100,000 policies requires 1 trillion random numbers
  - Run time measured in hours or even days

Monte Carlo simulation definitions and formulas

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- $face_j$ = face amount on policy j
- q_j = probability of a claim for policy j
- $CLAIM_j$ = random variable with a Bernoulli distribution with parameter q_j
  - Possible outcomes for $CLAIM_j$ are 0 (no claim on policy j) or 1 (claim on policy j)
  - Probability of a claim on policy j is q_j
  - Similar to modeling the outcome of a coin toss, except the probability is the expected mortality rate instead of 0.5

Monte Carlo simulation definitions and formulas

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1. Identify the claims for each trial

- $claim_{jk}$ = outcome of a simulation of the random variable $CLAIM_j$, where j is the policy and k is the trial number

2. Get the total claim amount for each trial

- $AggClaims_k$

  = the simulated total claim amount for a block of policies for trial k

  $= \sum_j \left(claim_{jk} \times face_j\right)$

3. Sort the trials to obtain percentiles

- All values of $AggClaims_k$ are sorted in increasing order so that percentiles can be obtained by selecting from the ordered values
  - With 10,000 trials, the 95th percentile would be approximately the 9500th value

Aggregate claims distribution: Two approaches

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Monte Carlo simulation

The Panjer method

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About the Panjer method

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- Recursive formula
- Based on a Poisson process
- Derived by Dr. Harry H. Panjer in his paper "The Aggregate Claims Distribution and Stop-Loss Reinsurance" (Transactions of the Society of Actuaries, Volume XXXII, 1980, pages 523-545)

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About the Panjer method

- Efficient approach
  - Run time measured in seconds
  - Increasing the number of trials in Monte Carlo simulations yields results that tend toward those from the Panjer method
- Formulas are recursive and summations use all combinations of amounts, making sample spreadsheet calculations difficult
- Need to "bucket" policies into face amount groups
- Implementation can be tricky since computers tend to "zero out" very small non-zero values

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The Panjer method definitions and formulas

Example:

Let U = $10,000

Let j = 25

Let n = 1,000

jU = $250,000

nU = $10 million

- U = the unit of insurance
- j = one unique issue amount category, measured in units of insurance
- n = the largest unique issue amount category, measured in units of insurance
- jU = the amount of insurance in issue amount category j, measured in dollars of insurance
- nU = the amount of insurance in the largest issue amount category, measured in dollars of insurance

The Panjer method definitions and formulas

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Example:

θ_j ≈ Expected # of $250k claims

X_j = total claims from all $250k policies

X = total claims from all policies of any amount

- θ_j = the sum of the forces of mortality for all lives in issue amount category j
- θ_j ≈ (Number of Policies in category j) * (Expected Mortality for category j), where Expected Mortality is a proxy for the force of mortality
- X_j = the random variable representing the aggregate claims of amount jU
- $X = X_1 + X_2 + ... + X_n$, the aggregate claims over all issue amount categories

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The Panjer method definitions and formulas

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Need to consider all possible combinations of claim amounts that could add up to iU

- P_i = Pr {X = iU} = the probability that the aggregate claims will be exactly iU

$$P_i = \left(1/i\right) \sum_{\substack{j=1 \\ \theta_i \neq 0}}^{min(i,n)} j\theta_j P_{i-j}$$

- In the special case where i=0 (probability of $0 in claims):

$$P_0 = Pr\{X = 0\} = exp\left(-\sum_{\substack{j=1 \\ \theta_i \neq 0}}^{n} \theta_j\right)$$

- The cumulative distribution function gives you the percentiles of the aggregate claims distribution

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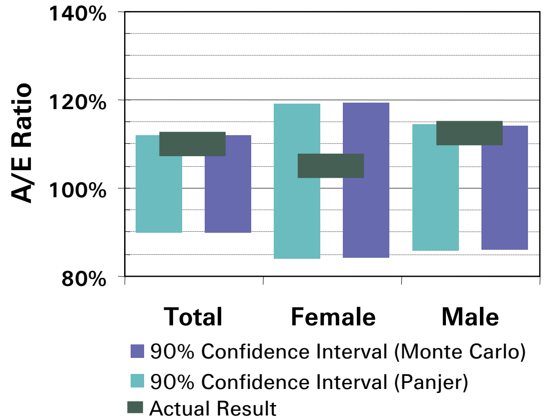
Samples of confidence intervals

- Compare actual-to-expected (A/E) ratios to confidence intervals centered at 100%



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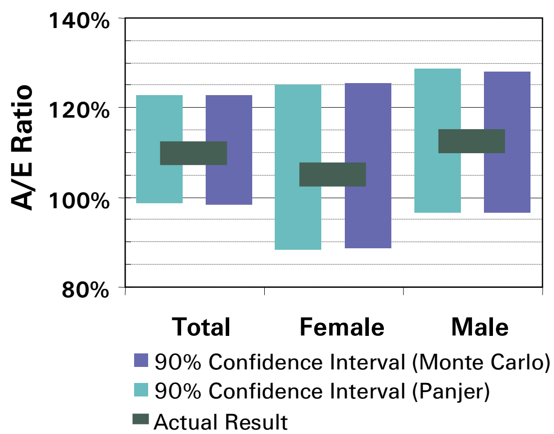
Samples of confidence intervals

- You can also center your CIs around the A/Es to illustrate volatility potential in experience



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ˌkre-də-ˈbi-lə-tē\ *noun*

- Definitions of credibility, according to www.merriam-webster.com:

1. The quality or power of inspiring belief
2. Capacity for belief

- Statistical definition, paraphrased from Thomas N. Herzog's Introduction to Credibility Theory:

  The application of one of several approaches to derive an estimate of the true value as a linear compromise between the current observations and the actuary's prior opinion

  $C = ZR + (1-Z)H$

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Credibility: a linear compromise

- General form of the credibility formula:

$$C = ZR + (1 - Z)H$$

  - C is the compromise estimator
  - R is the mean of the current observations
  - H is the prior mean
  - Z is the credibility factor

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Gain credibility

General formula:

$C = ZR + (1 - Z)H$

- Blend with a relevant industry table
  - e.g., H represents a value from the 2008 VBT
- Update your old assumptions using new experience
  - e.g., H is your old 20-year term duration 1 lapse rate assumption
- Principles-based reserving (PBR)
  - H will be prescribed

Volatility's influence on credibility

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General formula:

$C = ZR + (1-Z)H$

- Volatility of R (the experience) leads to volatility in the compromise estimate C
  - But is reduced depending on how small Z is
- Volatility inherent in H (the prior assumption) also leads to volatility in the compromise estimate C
  - But is reduced depending on how small Z is
  - Some credibility approaches help account for this

Credibility: Two common approaches

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Limited Fluctuation (Classical) approach

Bühlmann approach

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Credibility:
Two common approaches

Limited Fluctuation (Classical) approach

Bühlmann approach

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Limited Fluctuation approach

Square root formula has no theoretical basis, but it "feels right"

- General formula for Z:

$$Z = \min\left(\sqrt{\frac{actual\ \#\ claims}{\#\ claims\ needed\ \ for\ \ full\ credibility}}, 1\right)$$

- "# claims needed" is defined by formulas but inputs require actual judgment
  - It is the number of claims such that we are within a distance r of the true mean with probability p
  - Based on inverse of the Normal distribution
  - Example: To be within 5% (r) of the true mean with 90% (p) confidence, we need 1,083 claims
  - r and p are subjective

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Limited Fluctuation approach: By number vs. by amount

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- When working with by amount results, the "# claims needed" should be multiplied by a factor f:

$$\#\ claims\ needed\ (by\ amt) = \#\ claims\ needed\ (by\ number) \times f$$

- f factor accounts for variability in the amounts

$$f = \frac{average\ expected\ \left((claim\ amount)^2\right)}{(average\ expected\ claim\ amount)^2}$$

- The f factor ranges from about 2 to 5 for typical life insurance portfolios
- If all face amounts in the population are the same the f factor is 1 (can just use by number results)

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Limited Fluctuation approach: Selection of p and r

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- Number of claims needed for full credibility for various choices of p and r (f fixed at 3):

		# claims needed	
p	**r**	**by number**	**by amount (f=3)**
0.90	0.15	121	361
0.90	0.05	1,083	3,247
0.90	0.03	3,007	9,019
0.90	0.01	27,056	81,167
0.95	0.01	38,415	115,244
0.99	0.01	66,350	199,048

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Credibility: Two common approaches

Limited Fluctuation (Classical) approach

Bühlmann approach

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Bühlmann approach

- General formula for Z:

$$Z = \frac{n}{n+k}$$

  - n is the number of claims in the experience study
  - k is a factor related to the volatility of H (the prior mean):

    The relative standard deviation of H is $1/\sqrt{k}$

- Accounts for one weakness (volatility) in H

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Bühlmann approach: By number vs. by amount

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- A slight adjustment to the formula is needed when working with by amount results:

$$Z = \frac{n}{n + k \times f}$$

- The f factor is defined the same as before:

$$f = \frac{average\ expected\ \left((claim\ amount)^2\right)}{(average\ expected\ claim\ amount)^2}$$

Limited Fluctuation approach vs. Bühlmann approach

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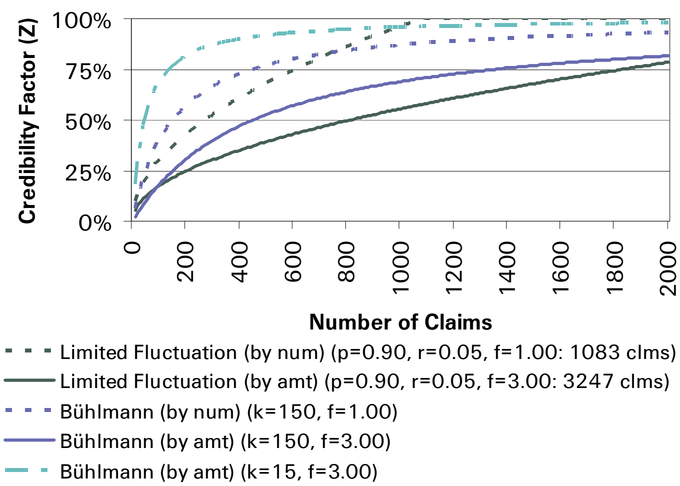
- Graphs of Z under various assumptions:



Additional considerations when applying credibility theory

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- Where to perform the credibility weighting
  - At the overall level
  - At the cell level
  - Combination
- Alternative (*H*) with which you're blending
  - Relevance
  - Recency
  - Volatility of *H*
- Actuarial judgment

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Further information on credibility

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- Thomas N. Herzog's textbook Introduction to Credibility Theory
- The American Academy of Actuaries' July 2008 Credibility Practice Note (http://www.actuary.org/pdf/practnotes/life_credibility08.pdf)
- Your favorite reinsurer

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Questions

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