Modeling Interest Rates and Inflation

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General

- Theory

  versus
- Practice

- Will be talking about generation of all kinds of economic scenarios, not just interest rates and inflation rates

Reasons to create scenario sets

- Regulatory requirements
  - SOP 03-01
  - RBC Phase 1
  - RBC Phase 2
- Product Pricing (e. g. VA Guar. Benefits)
- Derivative pricing exercises
  - FAS 133
  - Hedging programs

Reasons to create scenario sets

- Any questions so far?

Random Number Generator

- A good random number generator is at the heart of scenario generation
- Press, et al, say (page 214):

  “If all scientific papers whose results are in doubt because of bad Randoms were to disappear from library shelves, there would be a gap on each shelf about as big as your fist.”

Random Number Generator

- We've used Excel 97 to create some scenario sets.
- We also use a third party interest rate generator.
- We've also used the "pre-packaged" scenarios produced as part of the RBC C-3 Phase 2 process.

Random Number Generator

- Excel's random number generator
  (http://www.woodyswatch.com/office2003/archtemplate.asp?1-n11)

  "Microsoft reworked the function for Excel 2003 so that it would produce a better quality of randomness, especially when you ask for a large number of random numbers.

Random Number Generator

- “Guess what? If you do try to get a lot of random number, the function gets buggy, instead of numbers from 0 to 1 you start getting negative numbers. If that weren't enough, the negative numbers aren't sufficiently distributed to be considered 'random'.”

Random Number Generator

- The third party software we use allows 32,000 unique random seeds
- We create 1,000 scenarios, for 40 quarters, with two random numbers used for each quarter. So we use 1,000 x 40 x 2 = 80,000 of the 32,000 random numbers available

Random Number Generator

- The pre-packaged scenarios are only appropriate if the parameters used to generate them are appropriate to the task

Random Number Generator

- Any questions?

Inflation Scenarios

- Irving Fisher's theory:

  Short-term Treasury rate =

  risk free rate

  plus inflationary expectation
- Assure that a general model of inflation fits the purpose for which you are using it

Interest Rate Scenarios

- Arbitrage Free
  - absence of opportunities for risk-free profit
  - completeness of markets
  - relative prices that do not depend on individual investors' subjective views or risk preferences
  - expected-value pricing in risk neutral world

Interest Rate Scenarios

- "Real World"
  - Based on historical results or subjective opinion as to what the future might hold
  - Qualitative and quantitative stylized facts
    - Interest rates don't go to zero or infinity
    - Interest rates can spend years in a narrow range
    - Short-term and long-term rates are correlated, but not perfectly

Interest Rate Scenarios

- "Real World"
  - Stylized facts (continued)
    - Volatility of long-term rates is less than that of short-term rates
    - Yield curves have a variety of shapes
    - High volatility is associated with high interest rates
  - Estimate parameters from historical data.

Equity Scenarios

- Risk Neutral
  - All securities are assumed to earn the risk free rate
  - "Implied volatility" is estimated for each derivative, so as to duplicate its market value
- "Real World"
  - Based on historical results or subjective opinion as to what the future might hold

Equity Scenarios

- If doing a “real world” projection with derivatives, need to price the derivatives using risk neutral
- Can have major run-time issues with such a “stochastic on stochastic” projection
- See the Wendt article for one example of an historical approach
- Arithmetic vs. Geometric averages

Equity Scenarios

- There may be many kinds of equities which are appropriate to the task at hand - for example:
  - Domestic vs Foreign
  - Large vs Medium vs Small Capitalization
  - Value vs Growth
  - Equity positions in bond funds
  - Hedge funds

Equity Scenarios

- If modeling different kinds of equities, need to consider change in proportions as the model progresses.

Inflation, Interest, and Equity Scenarios

- Any questions?

How many scenarios?

- Always must ask “How many scenarios are needed for this purpose?”
- Generally need more scenarios if working with the tails than if working with the mean

How many scenarios?

- One approach is to run, say, 1,000 scenarios and calculate the statistic. Then run a second set of 1,000 scenarios. Is the statistic stable?
- If so, task complete.
- If not, combine the two sets. Run a third set. Is the statistic based on 3,000 scenarios the "same" as that based on 2,000?

How many scenarios?

- If so, task complete.
- If not, keep running additional sets of scenarios until the statistic stabilizes.

How many scenarios?

- Problems:
  - Don’t know at the start how many scenarios it will take.
  - Just because the statistic seems to stabilize doesn’t mean that it really has.

How many scenarios?

- Another approach is found in a paper by Manistre and Hancock
- They derive an estimate of the variance of the CTE estimator
- If the variance is “small”, probably enough scenarios have been used

How many scenarios?

- Any questions?

Run Time Considerations

- 12 minutes isn't much
- 1,000 times 12 minutes is 200 hours
- Brute force can work, if you have enough computer power available
- Christiansen and Chueh have published papers that may be useful

Run Time Considerations: Representative Scenarios

- Significance Method
  - Run all scenarios
  - Sort by result
  - Pick every n^{th} scenario

Run Time Considerations: Representative Scenarios

- Distance Method
  - Define “distance” (Easily said. How do you combine the effects of interest yield curves and different equity types?)
  - Pick an arbitrary scenario (Pivot 1 - P1)
  - Find distances from all other scenarios to P1 The one that is greatest becomes P2

Run Time Considerations Representative Scenarios

- Distance Measures (continued)
  - Find distances from all other scenarios to P1 and P2. The one that is furthest away from P1 and P2 becomes P3.
  - Continue until you have enough scenarios in your small set. Probability of each scenario in the set is determined by number of scenarios associated with it.

Run Time Considerations

- Problem with Significance Method:

  May not give enough points in the tails of the distribution
- Problem with Distance Method:

  Tends to over-sample the tails of the distribution

Run Time Considerations

- Low Discrepancy Methods
  - Tenney paper in Bibliography
- Other Variance Reduction Techniques
  - About 1.78 million Google hits, some of which may be relevant to some given task

Run Time Considerations

- Any questions?

Bibliography

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These are both on the SOA web site

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