Actuaries Club of the Southwest

June 11, 2009

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Financial Risk Management and Capital Optimization

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The Relationship Between the Two Concepts

- **Are they Automatically (Positively) Correlated?**
- **Or is it Possible to Have One Without the Other?**
- **Here is a Clue:**
  - **The purpose of financial risk management is to preserve and if possible enhance the financial integrity of a corporate entity.**

But what does this mean?

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'I Beg to Differ, Sir': Sometimes One, or the Other, or Neither

- But Which is Which?
- Example: Hedging VA 'GM' Riders

1. The classical risk management approach says delta-hedging is not enough.
2. The classical risk management approach says vega and gamma should also be hedged.
3. The classical risk management approach can get you into a lot of trouble.

Why? See Appendix I (on a night when you are having trouble falling asleep). The short version of Appendix I says that you could be doing everything the financial risk management discipline prescribes, and still impair the capital integrity of your company.

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'I Beg to Differ, Sir': Sometimes One, or the Other, or Neither

- Again, Which is Which?
- Another Example: ALM For Fixed Income Products
  1. We perform "duration" matching
  2. We even perform "convexity" matching
  3. What precisely does that do for us?
  4. Not much

Why? Duration and convexity concepts are built around the idea of "parallel shifts" in the yield curve.

A better alternative? The compact and very lucid text *Fixed Income Securities* by Robert Jarrow describes the no-arbitrage construct on which to base financial management of the fixed income liability (but I am not an agent or marketer for this book!)

The Heart of the Matter

But we digress. The important issue is providing financial risk management and capital optimization now, in the light of the crisis facing the financial world. How do we go about doing that?

First, Challenge Your Model

- Of course, We Are Assuming That You Do Have a Risk Model (Please Say Yes)
- There Are Hidden Capital Inefficiencies There That Need to Be Removed
  - Example 1: The three-currency circularity (this actually happened to someone I know)
  - Example 2: Ignoring correlation (or even deterministic parameter relationships)

1. Do you model mortality and longevity shocks independently and treat them as additive? Before you burst out laughing, understand that there are many companies doing just that
2. Similarly, do you treat inflation and deflation shocks as independent, ergo additive?

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Next, Curb Your Enthusiasm

- Implementing Requirements Before They Are Effective?
- Going Beyond Requirements (e.g. VAR at 99.99% When a Lower Percentile is Acceptable)?
- The Above and Similar Practices ……Should be Revisited and Revised Where Appropriate

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Take "Reasonable" Steps

- **Purchase Derivatives Directly From The Market to Hedge Your Product Optionalities**
- **But …….**
  1. Check the reduction in reserves versus option cost
  2. Have an internal model check the prices being quoted to you (ask me to elaborate)
- **Obtain Additional Risk-Transfer Capacity**
- Difficult
- **Change Your Product Design**
- Remove at-risk features (read "guarantees")
- Re-price those features whose risks have been underestimated
- **Shift Investment Allocation to Safer Instruments**
- But must be done in correlation with product re-design

Explore "Tongue in Cheek" Possibilities

- Government Relaxation of Capital Requirements
- Limit Premium Production of At-Risk Products
- Reduce commissions
- Set "current" = "guaranteed"
- Withdraw the product completely

Resort to “Desperate” Measures

- Raise New Capital
  - But in a depressed market, that is not the best idea
- Qualify Your Company as a Bank (No, Seriously)
  - Become eligible for TARP Funds
- Convert Debt to Equity (E.G. Preferred Shares)
  - Not entirely up to you

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Most Important, Conserve Your Cash

- It is Good to Focus on How Asset Cash Flows Affect Liability Structure
- Much More Important to Model How Liability Cash Flows Affect Asset Structure
- Joint Asset-Liability Dependency Model Required
- A Simplified Example is Shown in Appendix II

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… Appendix I…

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Stochastic Volatility

- **Stochastic Movement of Stock-Return Variance**

$$dV_t = V_t\,(\eta\, dt + \xi\, dW_t)$$

- **Volatility Defined**

$$\sigma_t = \sqrt{V_t}$$

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Stochastic Underlying

$$dS_t = S_t\,(r\,dt + \sigma_t\,dW_t)$$

Note: $\mathcal{W}_t$ and W_t are possibly correlated so what follows below is somewhat oversimplified

Current Value of Portfolio

The k functions represent the number of option contracts, and the h function represents the number of shares of the underlying.

$$P_t = -k_{j[t]}\,C_{j[t]} + k_{i[t]}\,C_{i[t]} + h_t\,S_t$$

Value of Portfolio a Small Time Increment Later

$$P_{(t+\Delta t)} = -k_{j[t]}\, C_{j[t+\Delta t]} + k_{i[t]}\, C_{i[t+\Delta t]} + h_t\, S_{(t+\Delta t)}$$

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Portfolio Change

Using appropriate Δ prefixes to represent change in portfolio values over small increment of time:

Note: This is symbolic only. The real mathematical expression of the portfolio change is given in the next section

$$\Delta P = -k_{j[t]}\, \Delta C_{j[t]} + k_{i[t]}\, \Delta C_{i[t]} + h_t\, \Delta S_t$$

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Apply Taylor Series Expansion to Second Power

$$
\begin{aligned}
\Delta P = & \left(-k_{j[t]}\, \partial_{S_t} C_{j[t]} + h_t + k_{i[t]}\, \partial_{S_t} C_{j[t]}\right) \Delta S_t + \\
& \left(-k_{j[t]}\, \partial_t C_{j[t]} + k_{i[t]}\, \partial_t C_{i[t]}\right) \Delta t + \left(-k_{j[t]}\, \partial_{V_t} C_{j[t]} + k_{i[t]}\, \partial_{V_t} C_{i[t]}\right) \Delta V_t + \\
& \left(-k_{j[t]}\, \partial_{S_t,S_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t,S_t} C_{i[t]}\right) \frac{1}{2} (\Delta S_t)^2 + \left(-k_{j[t]}\, \partial_{t,t} C_{j[t]} + k_{i[t]}\, \partial_{t,t} C_{i[t]}\right) \frac{1}{2} (\Delta t)^2 + \\
& \left(-k_{j[t]}\, \partial_{V_t,V_t} C_{j[t]} + k_{i[t]}\, \partial_{V_t,V_t} C_{i[t]}\right) \frac{1}{2} (\Delta V_t)^2 + \left(-k_{j[t]}\, \partial_{S_t,V_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t,V_t} C_{i[t]}\right) \Delta S_t\, \Delta V_t + \\
& \left(-k_{j[t]}\, \partial_{V_t,t} C_{j[t]} + k_{i[t]}\, \partial_{V_t,t} C_{i[t]}\right) \Delta V_t\, \Delta t + \left(-k_{j[t]}\, \partial_{t,S_t} C_{j[t]} + k_{i[t]\, \partial_{t,S_t} C_{i[t]}}\right) \Delta t\, \Delta S_t
\end{aligned}
$$

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Hedging Strategies

Note: Because only two options are involved, at most two hedging strategies can be implemented. The idea is to eliminate risk (variation) in the portfolio by reducing variation related to changes in the stochastic processes (i.e. ΔS and ΔV).

- **Delta**

Eliminate risk due to variation in underlying:

$$h_t - k_{j[t]}\, \partial_{S_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t} C_{i[t]} = 0$$

$$k_{i[t]} = 0$$

- **Delta-Gamma**

Eliminate risk due to variation in underlying as well as risk due to variation in square of underlying:

$$h_t - k_{j[t]}\, \partial_{S_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t} C_{i[t]} = 0$$

$$-k_{j[t]}\, \partial_{S_t,S_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t,S_t} C_{i[t]} = 0$$

- **Delta-Vega**

Eliminate risk due to variation in underlying as well as risk due to variation in volatility:

$$h_t - k_{j[t]}\, \partial_{S_t} C_{j[t]} + k_{i[t]}\, \partial_{S_t} C_{i[t]} = 0$$

$$-k_{j[t]}\, \partial_{V_t} C_{j[t]} + k_{i[t]}\, \partial_{V_t} C_{i[t]} = 0$$

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Special Note

It is dangerous to conclude that either delta-gamma or delta-vega hedging is necessarily an improvement on delta alone.

Notice that in delta-hedging, the only option that is brought into the equation is the one being hedged. In delta-gamma, the second option is brought in. But this introduces additional vega that is not hedged away. If the additional risk is greater than the risk reduced via delta-gamma, the portfolio is better off using delta alone.

A similar argument could be made for delta-vega.

Notice also that to accomplish delta, gamma and vega hedging simultaneously, an additional (third) option would have to be brought into the portfolio, complicating the residual risks even further.

… Appendix II …

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The aim is to determine the manner in which asset cash flows are dependent on emerging liability cash flows.

The model is developed without regard to any risk mitigating arrangements (such as mortality swaps).

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Joint Modeling of Assets And Liabilities

Assume that the total assets = 1 unit.

- **Step 1: The Probability That Assets Will Have to Be Liquidated**

The amount of cash assets (as a percentage of total) will be the major determinant of whether we will need to liquidate assets. Let us call it ζ.

The second determinant will be the nature of the liability. This is a rather difficult concept to quantify. In this example, it will be depicted as the mean liability plus two standard deviations. We will denote it by Ψ.

A final determinant could be the extent to which the issues of asset liability are closely monitored and managed. We will denote it by a factor (≤ 1) that multiplies both the above two determinants, and assign it the symbol $\mathcal{F}$.

We are now ready to describe the probability that assets will need to be liquidated.

Joint Modeling of Assets And Liabilities … con't

- **Beta Distribution**

Since we are at all times dealing with entities that are ≤ 1, the Beta distribution would be most appropriate to use. The distribution is first defined in terms of Ψ and $\mathcal{F}$ only.

Note: The β factor below is arbitrarily chosen. A tighter discipline needs to be developed around its determination.

```
β = .25;
Prob[Ξ_, F_] := BetaDistribution[F Ξ, β]
```

Here are some characteristics of the distribution:

```
Mean[Prob[Ξ, F]]
```

$$\frac{\mathcal{F}\,\Xi}{0.25+\mathcal{F}\,\Xi}$$

```
Variance[Prob[Ξ, F]]
```

$$\frac{0.25\,\mathcal{F}\,\Xi}{(0.25+\mathcal{F}\,\Xi)^2\,(1.25+\mathcal{F}\,\Xi)}$$

```
CDF[Prob[Ξ, F], x]
BetaRegularized[x, F Ξ, 0.25]
```

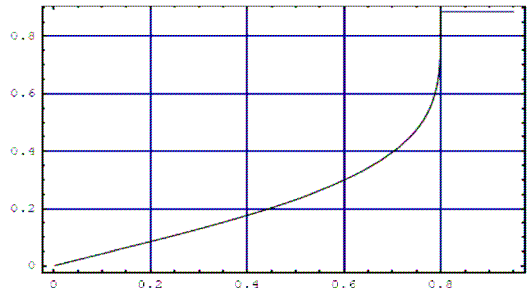
Joint Modeling of Assets And Liabilities … con't

- **Distribution of Asset Liquidation**

The probability that the amount of assets liquidated exceeds available cash by X units or less

```
P[X_, Φ_, F_, C_] := CDF[Prob[Φ, F], X + C] - CDF[Prob[Φ, F], C]

Plot[P[x, .9, .75, .2], {x, 0.0025, .95}, Frame → True, GridLines → Automatic]
```




```
- Graphics -
```

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The Probability That Liquidation Will Result in Loss

The determining factors will be:

(a) The liquidity of the assets. This will be denoted by a factor (≤ 1) that shows the ease with which assets can be converted to cash. We will assign it the symbol $\mathcal{L}$.

(b) The degree of asset/liability matching. Again, this will be denoted by a factor (≤ 1). It will multiply the liquidity factor above. Call this Φ.

A final determinant could be the relative maturity of the asset. We will denote it by a factor (≤ 1) that multiplies both the above two determinants, and assign it the symbol $\mathcal{F}$.

Note: The β factor below is arbitrarily chosen. A tighter discipline needs to be developed around its determination.

```
β = .25;

Pr[L_, Φ_, F_] := BetaDistribution[F Φ L, β]

Mean[Pr[L, Φ, F]]
```

$$\frac{\mathcal{F}\,\mathcal{L}\,\Phi}{0.25 + \mathcal{F}\,\mathcal{L}\,\Phi}$$

```
Variance[Pr[L, Φ, F]]
```

$$\frac{0.25\,\mathcal{F}\,\mathcal{L}\,\Phi}{(0.25 + \mathcal{F}\,\mathcal{L}\,\Phi)^2\,(1.25 + \mathcal{F}\,\mathcal{L}\,\Phi)}$$

```
Q[x_, L_, Φ_, F_] := CDF[Pr[L, Φ, F], x]
```

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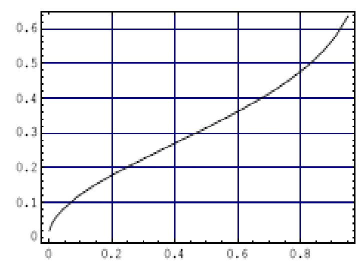
The Probability That Liquidation Will Result in Loss …con't.

```
Plot[Q[x, .9, .75, .75], {x, 0.0025, .95}, Frame → True, GridLines → Automatic]
```



- Graphics -

Copula

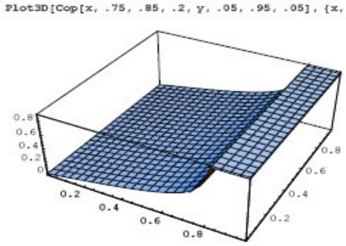
- **Gumbel Copula Parameter**

Note: This parameter is arbitrarily chosen. It needs to be supported by hard science

```
γ = 2.5;
```

- **Copula Formulation**

```
Cop[x_, Ξ_, ℱ_, C_, y_, ℒ_, Ξ_, θ_] := e^-((-Log[P[x,Ξ,ℱ,C]])^γ+(-Log[Q[y,ℒ,Ξ,θ]])^γ)^(1/γ)
Plot3D[Cop[x, .75, .85, .2, y, .05, .95, .05], {x, .0025, .995}, {y, .0025, .995}]
```



- SurfaceGraphics -